A Hydrodynamical Geometrization of Matter and Chronometricity in General Relativity

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Abstract: In this work, we outline a new complementary model of the relativistic theory of an inhomogeneous, anisotropic universe which was first very extensively proposed by Abraham Zelmanov to encompass all possible scenarios of cosmic evolution within the framework of the classical General Relativity, especially through the development of the mathematical theory of chronometric invariants. In doing so, we propose a fundamental model of matter as an intrinsic flexural geometric segment of the cosmos itself, i.e., matter is modelled as an Eulerian hypersurface of world-points that moves, deforms, and spins along with the entire Universe. The discrete nature of matter is readily encompassed by its representation as a kind of discontinuity curvature with respect to the background space-time. In addition, our present theoretical framework provides a very natural scheme for the unification of physical fields. An immediate scale-independent particularization of our preliminary depiction of the physical plenum is also considered in the form of an absolute monad model corresponding to a universe possessing absolute angular momentum.

Contents:

Dedicated to Abraham Zelmanov

§1. Introduction. The relativistic theory of a fully inhomogeneous, anisotropic universe in the classical framework of Einstein's General Theory of Relativity has been developed to a fairly unprecedented, over-arching extent by the general relativist and cosmologist Abraham Zelmanov [1]. The ingenious methodology of Zelmanov has been profoundly utilized and developed in several interesting physical situations, shedding further light on the intrinsic and extensive nature of the clas-

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sical General Relativity as a whole (see [2]).

By the phrase "classical General Relativity", we wish to emphasize that only space-time and gravitational fields have been genuinely, cohesively geometrized by the traditional Einsteinian theory. Nevertheless, the construction of the mathematical theory of chronometric invariants by Zelmanov enables one to treat General Relativity pretty much in the context of some kind of four-dimensional continuum mechanics of the very substance (plenum) of space-time geometry itself.

We factually note, in passing, that several independent theoretical approaches to the geometric unification of space-time, matter, and physical fields, in both the extensively classical sense and the non-classical sense, have been constructed by the Author elsewhere (for instance, see [3] and the bibliographical list of the Author's preceding works — diverse as they are — therein).

In the present work, we are singularly concerned with the methodology originally outlined by Zelmanov. Nevertheless, fully acquainted with the powerful depth, elegance, and beauty of his work, we shall still present some newly emerging ideas by first-principle construction, as well as some well-established understandings afresh, while uniquely situating ourselves in the alleyway wherefrom both the cosmos and the classical General Relativity are insightfully envisioned by Zelmanov.

As such, we shall theoretically fill a few gaps in the fabric of the classical General Relativity in general, and of Zelmanov's methodology in particular, by proposing a fundamental hydrodynamical model of matter, so as to possibly substantiate the material structure of the observer in common with the preferred, stable cosmic reference frame with respect to which the observer is at rest, i.e., one that co-moves, co-deforms, and co-rotates with respect to the entire Universe.

Indeed, we shall proceed first by geometrizing matter and discovering a natural way to reflectively superimpose the small-scale picture upon the entire Universe, yielding a unified description of the observer and the cosmos.

In the very general sense (far from the usual homogeneous, isotropic cosmological situations), we may note at this point that not all observers can automatically be qualified as fundamental observers, i.e. "observational monads", with respect to whose observation the structure of the Universe intrinsically appears the way it is observed by them.

Such, of course, is true also for observers assuming a homogeneous, isotropic universe and observing it accordingly. However, in certain cases incorporating, e.g., the absolute rotation of the universe, the problem of true interiority (and structural totality) arises in the sense that we can no longer recognize certain innate properties of the universe in the reference frames specific to homogeneous, isotropic models only. Such frames may be slowly translating and deforming to keep themselves at the natural expansive rate of the universe, but in the presence of self-rotation (intrinsic angular momentum), a physical system is quite something else to be accounted for in itself. For, if certain elementary particles (as we know them) are truly elementary, we shall know the total sense of observation of the Universe also from within their (common) interior and ultimately discover that, irrespective of scales, the Universe is self-contained in their very existence.

Now, recalling that which lies at the heart of the theory of chronometric invariants, we may posit further that the interior (and the total possible exterior) of the Universe can only be known by a rather advanced non-holonomic observer, i.e., one who is not merely "incidental" to the mesoscopic scale of (seemingly homogeneous) ordinary things, but one who builds his system of reference with respect to the interior and exterior of things in the required extreme limits, i.e., by rather direct in-depth cognition of the logically self-possible meta-Universe, beyond any self-limited experimental set-up. In other words, the totality of the laws of cognition is intrinsic to such an observer endowed with a "syntactical totality of logical operators" (a whole contingency of self-reflexive logical grammar). This, in turn, necessarily belongs to the interior of the directly observable (perceptual) Universe. One can then see how this substantially differs from a mere "bootstrap" universe.

Hence, regarding observation, our "anthropic principle with further self-qualification" is true only for observers dynamically situating themselves in certain unique non-holonomic frames of reference bearing the specific characteristics of motion of very elementary microscopic objects (such as certain elementary particles) and macroscopic objects exhibiting natural chronometricity with respect to the whole Universe (such as certain spinning stars, planets, galaxies, and metagalaxies). This, then, would be true for individual observers as well as an aggregate of common observers — such as those situated on a special rotating (planetary) islet of mass — in their own unique ("universally preferred") non-holonomic coordinate systems.

Such observers are truly situated at the world-points of the respective Eulerian hypersurfaces (representing matter) in common with the entire non-holonomic, inhomogeneous, anisotropic Universe. This is because, while "inhering" in matter itself, they automatically possess all the geometric material configurations intrinsic to both matter itself and the entire Universe.

In our theory, as we shall see, the projective chronometric structure plays the role of the geometrized non-Abelian gauge field strength. Hence, any natural extension of the study will center around the corresponding emphasis that the inner constitution of elementary particles cannot be divorced from chronometricity (the way it is hydrodynamically geometrized here). This, like in the original case of Zelmanov, gives one the penetrating confidence to speak of elementary particles, in addition to large-scale objects, purely in the framework of the chronometric General Relativity, as if without having to mind the disparities involving scales (of particles and galaxies).

Here, chronometricity is geometrized in such a way that co-substantial motion results, both hydrodynamically and geometrically, from the fundamental properties of the extrinsic curvature of the material hypersurface (i.e., matter itself).

In addition, the Yang-Mills curvature is generalized by the presence of the asymmetric extrinsic curvature, as in [5]. Only in pervasive flatness does it go into the usual Yang-Mills form of the Standard Model (whose background space-time is Minkowskian). We shall not employ the full form of the particular Finslerian connection as introduced in [3], but only the respective metric-compatible part, with the corresponding geodesic equation of motion intrinsically generating the generally covariant Lorentz equation.

Hence, while encompassing the elasticity of space-time, we shall further advance the notion of a discontinuous Eulerian hypersurface such that it geometrically represents matter and chronometricity at once, and such that it may be applied to any cosmological situation independently of scales.

Indeed, as we shall see, the Machian construction (see §5 herein) is a special condition for "emergent inertia", without having to invoke both Newtonian absolute (external) empty space and a distant reference frame. Rather, the whole process is meant to be topologically scale-independent. An alternative objective of the present approach, therefore, is such that the structure of General Relativity, when developed (generalized) this way, can apparently meet that of quantum theory in a parallel fashion.

§2. The proposed geometrization of matter: a cosmic monad. Let us consider an arbitrary orientation of a mobile, spinning hypersurface $\mathrm{C}^3(t) = \partial\Sigma^4$ as the boundary of the world-tube Σ^4 of geodesics in the background Universe M^4. Denoting the regular boundary by $\mathrm{B}^3(t)$ and the discontinuity hypersurface cutting through Σ^4 by Υ, we see that $\mathrm{C}^3(t) = \mathrm{B}^3(t) \cup \Upsilon$. We emphasize that $\mathrm{C}^3(t)$ is a natural geometric segment of M^4, i.e., it is created purely by the dynamics of the intrinsic (and global) curvature and torsion of M^4. This is to the extent that the unit normal vector with respect to $\mathrm{C}^3(t)$ is immediately given by the world-velocity $u^\alpha(s)$ along the world-line s.

We call $\mathrm{C}^3(t)$ a monad, i.e., a substantive Eulerian structure of matter. As we shall see, this dynamical monad model is fully intrinsic to the fabric of space-time, i.e., inseparable from (not external to) the intrinsic structure of the Universe, thereby allowing us to incorporate the subsequent geometrization of matter (and material fields) into Einstein's field equation.

Our substantial depiction of matter filling the cosmos also implies the wave-like nature of the hypersurface $\mathrm{C}^3(t)$, for the velocity field of the points of $\mathrm{C}^3(t)$ — representing individual group particles — is no longer singly oriented. This allows us to project the fundamental material structure pervasively outward — onto the Universe itself. Consequently, this model readily applies to all sorts of observers, other than just a comoving one (whose likeness we shall especially refer to as the "purely monad observer").

As we know, the infinitesimal world-line, along which $\mathrm{C}^3(t)$ moves, is explicitly given by the metric tensor $g_{\alpha\beta}(x)$ of M^4 as

$$\begin{aligned} ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = g_{00}\, dx^0 dx^0 + 2\, g_{0i}\, dx^0 dx^i + g_{ik}\, dx^i dx^k = \\ = c^2 d\tau^2 - d\sigma^2, \end{aligned} \tag{2.1}$$

where we denote the speed of light as c. The proper time, the generally non-holonomic, evolutive spatial segment (the hypersurface segment), the metric tensor of the hypersurface, and the linear velocity of space rotation (i.e., of material spin) are respectively given by*

$$d\tau = \frac{g_{0\alpha}\, dx^\alpha}{c\sqrt{g_{00}}} = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i, \tag{2.2}$$

$$d\sigma = \sqrt{h_{ik}\, dx^i dx^k}\,, \tag{2.3}$$

$$h_{ik} = -\, g_{ik} + \frac{g_{0i}\, g_{0k}}{g_{00}} = -\, g_{ik} + \frac{1}{c^2}\, v_i v_k\,, \tag{2.4}$$

$$v_i = -\, c\, \frac{g_{0i}}{\sqrt{g_{00}}}\,. \tag{2.5}$$

*Einstein's summation convention is utilized with space-time Greek indices running from 0 to 3 and projective material-spatial Latin indices from 1 to 3.

Denoting the unit normal vector of the material hypersurface by N^α, we see that $N^\alpha = u^\alpha = \frac{dx^\alpha}{ds}$, and especially that

$$N_i = u_i = -\frac{1}{c}\, v_i\,. \tag{2.6}$$

In a simplified matrix representation, we therefore have

$$g_{\alpha\beta} = \begin{pmatrix} g_{00} & N_i\sqrt{g_{00}} \\ N_i\sqrt{g_{00}} & -h_{ik} + N_i N_k \end{pmatrix}. \tag{2.7}$$

Now, the fundamental projective relation between the background space-time metric $g_{\alpha\beta}(x)$ and the global material metric $h_{ik}(x,u)$ is readily given as

$$g_{\alpha\beta} = -h_{\alpha\beta} + u_\alpha u_\beta\,, \tag{2.8}$$

where, with $f\,(v^i, dt) \to v^i f\,(dt)$ and $f\big(v_i, \frac{\partial}{\partial t}\big) \to v_i\, f\big(\frac{\partial}{\partial t}\big)$,

$$h_{\alpha\beta} = \frac{\partial Y^i}{\partial x^\alpha}\frac{\partial Y^k}{\partial x^\beta}\,(-\,g_{ik})\,, \tag{2.9}$$

$$dY^i = dx^i + f\,(v^i, dt)\,, \tag{2.10}$$

$$\frac{\partial}{\partial Y^i} = \frac{\partial x^\alpha}{\partial Y^i}\frac{\partial}{\partial x^\alpha} = \frac{\partial}{\partial x^i} + f\left(v_i, \frac{\partial}{\partial t}\right), \tag{2.11}$$

$$h^\alpha_\beta = -\,\delta^\alpha_\beta + u^\alpha u_\beta\,, \qquad h^\alpha_\gamma h^\gamma_\beta = \delta^\alpha_\beta - u^\alpha u_\beta\,, \tag{2.12}$$

$$h^i_\alpha = h^\beta_\alpha \frac{\partial Y^i}{\partial x^\beta} = -\frac{\partial Y^i}{\partial x^\alpha}\,, \tag{2.13}$$

$$h^\alpha_i h^i_\beta = \delta^\alpha_\beta - u^\alpha u_\beta\,, \qquad h^i_\alpha h^\alpha_k = \delta^i_k\,, \tag{2.14}$$

$$h_{\alpha\beta}\, u^\beta = 0\,, \qquad h^i_\alpha\, u^\alpha = 0\,. \tag{2.15}$$

Let us represent the natural basis vector of M^4 by $\bar g_\alpha$ and that of $\mathrm{C}^3(t)$ by $\bar\omega_i$. We immediately obtain the generally asymmetric extrinsic curvature of $\mathrm{C}^3(t)$ through the inner product

$$Z_{ik} = \left\langle u, \frac{\partial \bar\omega_i}{\partial Y^k}\right\rangle \tag{2.16}$$

i.e.,

$$Z_{ik} = -\,u_\alpha \nabla_k\, h^\alpha_i = h^\alpha_i\, h^\beta_k\, \nabla_\beta\, u_\alpha\,, \tag{2.17}$$

where ∇ denotes covariant differentiation, i.e., for an arbitrary tensor field $Q^{ab\dots}_{cd\dots}(x)$ and metric-compatible connection form $\Gamma^a_{mk}(x)$, presented herein with arbitrary indexing,

$$\nabla_k Q^{ab\dots}_{cd\dots} = \frac{\partial Q^{ab\dots}_{cd\dots}}{\partial x^k} + \Gamma^a_{mk} Q^{mb\dots}_{cd\dots} + \Gamma^b_{mk} Q^{am\dots}_{cd\dots} + \dots - \Gamma^m_{ck} Q^{ab\dots}_{md\dots} - \Gamma^m_{dk} Q^{ab\dots}_{cm\dots} - \dots\,, \tag{2.18}$$

$$\Gamma^a_{bc} = \frac{1}{2}\, g^{am} \left(\frac{\partial g_{mb}}{\partial x^c} - \frac{\partial g_{bc}}{\partial x^m} + \frac{\partial g_{cm}}{\partial x^b} \right) + \Gamma^a_{[bc]} - g^{am} \left(g_{bn} \Gamma^n_{[mc]} + g_{cn} \Gamma^n_{[mb]} \right), \tag{2.19}$$

$$\frac{\mathrm{D} Q^{ab\dots}_{cd\dots}}{ds} = u^e\, \nabla_e\, Q^{ab\dots}_{cd\dots}\,. \tag{2.20}$$

Henceforth, round and square brackets on indices shall indicate symmetrization and anti-symmetrization, respectively.

Hence, we see that the extrinsic curvature tensor of the material hypersurface is uniquely expressed in terms of the four-dimensional velocity gradient tensor given by the expression

$$\varphi_{\alpha\beta} = \nabla_\beta\, u_\alpha\,, \tag{2.21}$$

i.e.,

$$Z_{ik} = h^\alpha_i\, h^\beta_k\, \varphi_{\alpha\beta}\,. \tag{2.22}$$

This way, we have indeed geometrized the tensor of the rate of material deformation $\Theta_{\alpha\beta}$ and the tensor of material vorticity $\omega_{\alpha\beta}$, as can be seen from the respective symmetric and anti-symmetric expressions below:

$$Z_{(ik)} = h^\alpha_i\, h^\beta_k\, \Theta_{\alpha\beta}\,, \qquad Z_{[ik]} = h^\alpha_i\, h^\beta_k\, \omega_{\alpha\beta}\,, \tag{2.23}$$

where

$$\Theta_{\alpha\beta} = \frac{1}{2} \left(\nabla_\beta\, u_\alpha + \nabla_\alpha\, u_\beta \right), \qquad \omega_{\alpha\beta} = \frac{1}{2} \left(\nabla_\beta\, u_\alpha - \nabla_\alpha\, u_\beta \right). \tag{2.24}$$

Meanwhile, noting immediately that

$$\nabla_k\, h^\alpha_i = -\, Z_{ik}\, u^\alpha, \tag{2.25}$$

we obtain the following relation:

$$\frac{\partial h^\alpha_i}{\partial Y^k} = \Omega^p_{ik}\, h^\alpha_p - \Gamma^\alpha_{\beta\gamma}\, h^\beta_i\, h^\gamma_k - Z_{ik}\, u^\alpha. \tag{2.26}$$

Both $\Omega^p_{ik}(Y^p)$ of $\mathrm{C}^3(t)$ and $\Gamma^\alpha_{\beta\gamma}(x)$ of M^4 are generally asymmetric, non-holonomic connection forms. We see that they are related to each other through the following fundamental relations:

$$\Omega^{p}_{ik} = h^{p}_{\alpha}\,\frac{\partial h^{\alpha}_{i}}{\partial Y^{k}} - h^{p}_{\alpha}\,\Gamma^{\alpha}_{\beta\gamma}\,h^{\beta}_{i}\,h^{\gamma}_{k}\,, \tag{2.27}$$

$$\Gamma^{\alpha}_{\beta\gamma} = h^{\alpha}_{i}\,\frac{\partial h^{i}_{\beta}}{\partial x^{\gamma}} - h^{\alpha}_{p}\,\Omega^{p}_{ik}h^{i}_{\beta}h^{k}_{\gamma} + Z_{ik}\,h^{i}_{\beta}h^{k}_{\gamma}u^{\alpha} + u^{\alpha}\,\frac{\partial u_{\beta}}{\partial x^{\gamma}} - Z^{i}_{\cdot k}\,h^{\alpha}_{i}\,h^{k}_{\gamma}u_{\beta}\,. \tag{2.28}$$

The associated curvature tensor of $\mathrm{C}^{3}(t)$, $R^{i\cdot\cdot\cdot}_{\cdot jkl}(\Omega^{i}_{jl})$, and that of M^{4}, $R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\rho\gamma}(\Gamma^{\alpha}_{\beta\gamma})$, are then respectively given by

$$R^{i\cdot\cdot\cdot}_{\cdot jkl} = \frac{\partial\Omega^{i}_{jl}}{\partial Y^{k}} - \frac{\partial\Omega^{i}_{jk}}{\partial Y^{l}} + \Omega^{p}_{jl}\,\Omega^{i}_{pk} - \Omega^{p}_{jk}\,\Omega^{i}_{pl}\,, \tag{2.29}$$

$$R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\rho\gamma} = \frac{\partial\Gamma^{\alpha}_{\beta\gamma}}{\partial x^{\rho}} - \frac{\partial\Gamma^{\alpha}_{\beta\rho}}{\partial x^{\gamma}} + \Gamma^{\tau}_{\beta\gamma}\Gamma^{\alpha}_{\tau\rho} - \Gamma^{\tau}_{\beta\rho}\Gamma^{\alpha}_{\tau\gamma}\,, \tag{2.30}$$

where, as usual,

$$\begin{aligned}\big(\nabla_{l}\nabla_{k} - \nabla_{k}\nabla_{l}\big)Q^{ab\ldots}_{cd\ldots} &= \\ &= R^{m\cdot\cdot\cdot}_{\cdot ckl}Q^{ab\ldots}_{md\ldots} + R^{m\cdot\cdot\cdot}_{\cdot dkl}Q^{ab\ldots}_{cm\ldots} + \cdots - R^{a\,\cdot\cdot\cdot}_{\cdot mkl}Q^{mb\ldots}_{cd\ldots} - \\ &\quad - R^{b\,\cdot\cdot\cdot}_{\cdot mkl}Q^{am\ldots}_{cd\ldots} - \cdots - 2\Gamma^{m}_{[kl]}\nabla_{m}\,Q^{ab\ldots}_{cd\ldots}\,,\end{aligned} \tag{2.31}$$

$$\big(\nabla_{b}\nabla_{a} - \nabla_{a}\nabla_{b}\big)\,\phi = -2\Gamma^{c}_{[ab]}\,\nabla_{c}\,\phi\,, \tag{2.32}$$

where ϕ is an arbitrary scalar field.

At this point, we obtain the complete projective relations between the background space-time geometry and the geometric material space. The relations are as follows:

$$R_{ijkl} = Z_{ik}\,Z_{jl} - Z_{il}\,Z_{jk} + h^{\alpha}_{i}\,h^{\beta}_{j}\,h^{\rho}_{k}\,h^{\gamma}_{l}\,R_{\alpha\beta\rho\gamma} + S_{\alpha jkl}\,h^{\alpha}_{i}\,, \tag{2.33}$$

$$\nabla_{l}\,Z_{ik} - \nabla_{k}\,Z_{il} = u^{\alpha}\,h^{\beta}_{i}\,h^{\rho}_{k}\,h^{\gamma}_{l}\,R_{\alpha\beta\rho\gamma} - 2\,\Omega^{p}_{[kl]}\,Z_{ip} + u^{\alpha}\,S_{\alpha ikl}\,. \tag{2.34}$$

In terms of the curvature tensor $R^{i\cdot\cdot\cdot}_{\cdot jkl}(\Omega^{i}_{jl})$ of $\mathrm{C}^{3}(t)$, and that of M^{4}, which is $R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\rho\gamma}(\Gamma^{\alpha}_{\beta\gamma})$, with the segmental torsional curvature (incorporating possible analytical discontinuities as well) given by

$$S^{\alpha\cdot\cdot\cdot}_{\cdot ijk} = \frac{\partial}{\partial Y^{j}}\left(\frac{\partial h^{\alpha}_{i}}{\partial Y^{k}}\right) - \frac{\partial}{\partial Y^{k}}\left(\frac{\partial h^{\alpha}_{i}}{\partial Y^{j}}\right) + \Gamma^{\alpha}_{\beta\gamma}\,h^{\beta}_{i}\left(\frac{\partial h^{\gamma}_{j}}{\partial Y^{k}} - \frac{\partial h^{\gamma}_{k}}{\partial Y^{j}}\right). \tag{2.35}$$

Now, we can four-dimensionally express the (generalized generally covariant) gravitational force F_{α}, the spatial deformation $D_{\alpha\beta}$, and the angular momentum $A_{\alpha\beta}$ in terms of our geometrized material deformation and material vorticity as follows:

$$F_\alpha = 2c^2 u^\beta \omega_{\alpha\beta}\,, \tag{2.36}$$

$$D_{\alpha\beta} = c h^\mu_\alpha h^\nu_\beta \Theta_{\mu\nu}\,, \tag{2.37}$$

$$A_{\alpha\beta} = c h^\mu_\alpha h^\nu_\beta\, \omega_{\mu\nu}\,, \tag{2.38}$$

such that

$$\Phi_{\alpha\beta} = c h^\mu_\alpha h^\nu_\beta\, \varphi_{\mu\nu} = D_{\alpha\beta} + A_{\alpha\beta}\,, \tag{2.39}$$

$$D_{ik} = h^\mu_i h^\nu_k \Phi_{(\mu\nu)}\,, \tag{2.40}$$

$$A_{ik} = h^\mu_i h^\nu_k \Phi_{[\mu\nu]}\,. \tag{2.41}$$

As a result, we obtain the geometrized dynamical relation

$$R_{ijkl} = h^\alpha_i h^\beta_j h^\rho_k h^\gamma_l \left(R_{\alpha\beta\rho\gamma} + \varphi_{\alpha\rho}\varphi_{\beta\gamma} - \varphi_{\alpha\gamma}\varphi_{\beta\rho}\right) + h^\alpha_i S_{\alpha jkl}\,. \tag{2.42}$$

Furthermore, let us introduce the Eulerian (substantive) curvature of the material hypersurface which satisfies all the natural symmetries of the curvature and torsion tensors of the background space-time as follows:

$$F_{ijkl} = h^\alpha_i h^\beta_j h^\rho_k h^\gamma_l R_{\alpha\beta\rho\gamma} + h^\alpha_i S_{\alpha jkl}\,. \tag{2.43}$$

We immediately see that

$$F_{ijkl} = R_{ijkl} - \frac{1}{c}\big(D_{ik}D_{jl} - D_{il}D_{jk} + A_{ik}A_{jl} - A_{il}A_{jk} + {} + D_{ik}A_{jl} - D_{il}A_{jk} + A_{ik}D_{jl} - A_{il}D_{jk}\big)\,, \tag{2.44}$$

and, in addition, we also obtain the inverse projective relations

$$R_{\mu\nu\rho\sigma} - R_{\lambda\nu\rho\sigma}u^\lambda u_\mu - R_{\mu\lambda\rho\sigma}u^\lambda u_\nu - R_{\mu\nu\lambda\sigma}u^\lambda u_\rho - {} - R_{\mu\nu\rho\lambda}u^\lambda u_\sigma - R_{\lambda\nu\kappa\sigma}u^\lambda u^\kappa u_\mu u_\rho - R_{\lambda\nu\rho\kappa}u^\lambda u^\kappa u_\mu u_\sigma - {} - R_{\mu\lambda\kappa\sigma}u^\lambda u^\kappa u_\nu u_\rho - R_{\mu\lambda\rho\kappa}u^\lambda u^\kappa u_\nu u_\sigma = {} = h^j_\nu h^k_\rho h^l_\sigma\big(h^i_\mu(R_{ijkl} - Z_{ik}Z_{jl} + Z_{il}Z_{jk}) + S_{\mu jkl} - u_\mu u^\lambda S_{\lambda jkl}\big)\,, \tag{2.45}$$

$$R_{\lambda\mu\nu\rho}u^\lambda - R_{\lambda\mu\kappa\rho}u^\lambda u^\kappa u_\nu - R_{\lambda\mu\nu\kappa}u^\lambda u^\kappa u_\rho = {} = -h^i_\mu h^k_\nu h^l_\rho\big(\nabla_l Z_{ik} - \nabla_k Z_{il} + 2\Omega^p_{[kl]}Z_{ip} - S_{\lambda ikl}u^\lambda\big)\,. \tag{2.46}$$

The complete geometrization of matter in this hydrodynamical approach represents a continuum mechanical description of space-time where the extrinsic curvature of any material hypersurface manifests itself as the gradient of its velocity field. As such, the geometric field equations simply consist in specifying the world-velocity of the moving matter (especially directly from reading off the components of the fundamental metric tensor). The acquisition of individual particles, as a special case of the more general group particles, is immediately at hand when the material hypersurface enclosing a volumetric segment of the cosmos is small enough, i.e., in this case the particles are ordinary infinitesimal space-time points translating and spinning in common with the deforming and spinning Universe on the largest scale.

§3. Reduction to the pure monad model. Having formulated the general structure of our scheme for the substantive geometrization of matter (as well as physical fields, essentially by way of our preceding works as listed in [3]) in the preceding section, we can now explicitly arrive at the cosmological picture of Zelmanov for general relativistic dynamics, i.e., the theory of chronometric invariants.

Much in parallel with Yershov [4], we may simply state the strong monad model of the cosmos of Zelmanov as follows:

1) The Universe as a whole spins, inducing the spin of every elementary constituent in it;
2) The Universe is intrinsically inhomogeneous, anisotropic, and nonholonomic, giving rise to its diverse elementary constituents (i.e., particles) on the microscopic scale, including its specific fundamental properties (e.g., mass, charge, and spin);
3) The small-scale structure of the Universe is simply holographic ("isomorphic") to the large-scale cosmological structure, thereby rendering the Universe truly self-contained;
4) The linear velocity (or momentum) of any microscopic or macroscopic object is essentially induced by the global spin of the Universe, such that the individual motion of matter is none other than the segmental motion of the Universe.

We shall refer to the above conventions as the *pure monad model*.

Consequently, we have the chronometrically invariant condition represented by

$$f\left(v^i, dt\right) = 0\,, \tag{3.1}$$

$$f\left(v_i, \frac{\partial}{\partial t}\right) \neq 0\,. \tag{3.2}$$

Therefore, with respect to the material hypersurface $\mathrm{C}^3(t)$, we see that

$$h_{ik} = -\,g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \tag{3.3}$$

$$h^{ik} = -\,g^{ik}, \qquad h^i_\alpha = -\,\delta^i_\alpha\,, \tag{3.4}$$

$$v_i = -\,c\,\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad v^i = -\,c\sqrt{g_{00}}\;g^{0i}, \tag{3.5}$$

$$v^2 = v_i v^i = h_{ik}\,v^i v^k\,, \tag{3.6}$$

$$u_0 = \sqrt{g_{00}}\,, \qquad u^0 = \frac{1}{\sqrt{g_{00}}}\,, \tag{3.7}$$

$$u_i = \frac{g_{0i}}{\sqrt{g_{00}}} = -\frac{v_i}{c}\,, \qquad u^i = 0\,, \tag{3.8}$$

$$dY^i = dx^i. \tag{3.9}$$

In our theory, Zelmanov's usual differential operators of chronometricity are given by

$$\frac{\partial}{\partial Y^i} = \frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\,v_i\,\frac{{}^*\partial}{\partial t}\,, \tag{3.10}$$

$$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}\,, \tag{3.11}$$

$$\frac{\partial}{\partial Y^i}\frac{{}^*\partial}{\partial t} - \frac{{}^*\partial}{\partial t}\frac{\partial}{\partial Y^i} = \frac{{}^*\partial^2}{\partial x^i\,\partial t} - \frac{{}^*\partial^2}{\partial t\,\partial x^i} = \frac{1}{c^2}\,F_i\,\frac{{}^*\partial}{\partial t}\,, \tag{3.12}$$

$$\frac{\partial^2}{\partial Y^k\,\partial Y^i} - \frac{\partial^2}{\partial Y^i\partial Y^k} = \frac{{}^*\partial^2}{\partial x^k\,\partial x^i} - \frac{{}^*\partial^2}{\partial x^i\,\partial x^k} = -\frac{2}{c^2}\,A_{ik}\,\frac{{}^*\partial}{\partial t}\,. \tag{3.13}$$

Here the three-dimensional gravitational-inertial force, material deformation, and angular momentum are simply given by the three-dimensional chronometrically invariant components of the four-dimensional quantities F_α, $D_{\alpha\beta}$, and $A_{\alpha\beta}$ of the preceding §2 — in the case of vanishing background torsion — as follows:

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \tag{3.14}$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \tag{3.15}$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right), \tag{3.16}$$

where the associated Zelmanov identities are

$$\frac{\partial A_{ik}}{\partial Y^l} + \frac{\partial A_{kl}}{\partial Y^i} + \frac{\partial A_{li}}{\partial Y^k} = \frac{{}^*\partial A_{ik}}{\partial x^l} + \frac{{}^*\partial A_{kl}}{\partial x^i} + \frac{{}^*\partial A_{li}}{\partial x^k} = \\ = -\frac{1}{c^2}\left(A_{ik}F_l + A_{kl}F_i + A_{li}F_k\right), \tag{3.17}$$

$$\frac{{}^*\partial A_{ik}}{\partial t} = \frac{1}{2}\left(\frac{\partial F_i}{\partial Y^k} - \frac{\partial F_k}{\partial Y^i}\right) = \frac{1}{2}\left(\frac{{}^*\partial F_i}{\partial x^k} - \frac{{}^*\partial F_k}{\partial x^i}\right), \tag{3.18}$$

and the gravitational potential scalar is

$$\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right). \tag{3.19}$$

The symmetric chronometrically invariant connection of Zelmanov can be given here by

$$\Delta^i_{kl} = \Omega^i_{(kl)} + h^{ip}\left(h_{kq}\Omega^q_{[pl]} + h_{lq}\Omega^q_{[pk]}\right) = \frac{1}{2}\,h^{ip}\left(\frac{\partial h_{pk}}{\partial Y^l} - \frac{\partial h_{kl}}{\partial Y^p} + \frac{\partial h_{lp}}{\partial Y^k}\right) = \\ = \frac{1}{2}\,h^{ip}\left(\frac{{}^*\partial h_{pk}}{\partial x^l} - \frac{{}^*\partial h_{kl}}{\partial x^p} + \frac{{}^*\partial h_{lp}}{\partial x^k}\right) = \\ = \frac{1}{2}\,h^{ip}\left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p} + \frac{\partial h_{lp}}{\partial x^k}\right) + \frac{1}{c^2}\left(D^i_k v_l - D_{kl}v^i + D^i_l v_k\right). \tag{3.20}$$

Note that while the extrinsic curvature tensor Z_{ik} is naturally asymmetric in our theory (in order to account for geometrized material vorticity), we might impose symmetry upon the material connection Ω^p_{ik} whenever convenient (or else we can associate its anti-symmetric part, through projection with respect to the background torsion, with the electromagnetic and chromodynamical gauge fields, as we have done, e.g., in [3] and [5]).

Now, with respect to the geometrized dynamical relations of the preceding section, we obtain

$$R_{\mu\nu\rho\sigma} = h^j_\nu h^k_\rho h^l_\sigma\left(h^i_\mu(R_{ijkl} - Z_{ik}Z_{jl} + Z_{il}Z_{jk}) + S_{\mu jkl} - u_\mu u^\lambda S_{\lambda jkl}\right) + \\ + u_\mu X_{\nu\rho\sigma} - u_\nu X_{\mu\rho\sigma} + u_\rho Y_{\mu\nu\sigma} - u_\sigma Y_{\mu\nu\rho} + u_\mu u_\rho J_{\nu\sigma} - u_\mu u_\sigma J_{\nu\rho} + \\ + u_\nu u_\rho K_{\mu\sigma} - u_\nu u_\sigma K_{\mu\rho}, \tag{3.21}$$

where

$$X_{\alpha\beta\gamma} = \frac{R_{0\alpha\beta\gamma}}{\sqrt{g_{00}}}, \qquad Y_{\alpha\beta\gamma} = \frac{R_{\alpha\beta 0\gamma}}{\sqrt{g_{00}}}, \tag{3.22}$$

$$J_{\alpha\beta} = \frac{R_{0\alpha 0\beta}}{g_{00}}, \qquad K_{\alpha\beta} = \frac{R_{\alpha 00\beta}}{g_{00}} = -J_{\alpha\beta}. \tag{3.23}$$

These quantities, whose three-dimensional components may be linked with Zelmanov's various three-dimensional curvature tensors [1, 2], appear to correspond to certain generalized currents.

Further calculation reveals that

$$X_{\mu\nu\rho} + u_\nu J_{\mu\rho} - u_\rho J_{\mu\nu} = \\ = -h^i_\mu h^k_\nu h^l_\rho \left(\nabla_l Z_{ik} - \nabla_k Z_{il} + 2\Omega^p_{[kl]} Z_{ip} - S_{\lambda ikl}\, u^\lambda\right), \quad (3.24)$$

$$Y_{\mu\nu\rho} = \frac{2\sqrt{g_{00}}}{\sqrt{g_{00}} - 1}\left(u_\nu J_{\mu\rho} - u_\mu J_{\nu\rho}\right). \quad (3.25)$$

Therefore, the immediate general significance of these currents lies in the dynamical formation of matter itself with respect to the background structure of the world-geometry (represented by M^4).

§4. Hydrodynamical unification of physical fields. In this section, we shall deal with the explicit structure of the connection form underlying the world-manifold M^4, as well as that of matter — the material hypersurface $\mathrm{C}^3(t)$, by recalling certain fundamental aspects of our particular approach to the geometric unification of physical fields outlined in [3] and [5], which very naturally gives us the correct equation of motion for a particle (endowed with structure) moving in gravitational and electromagnetic fields while internally also experiencing the Yang-Mills gauge field, i.e., as an intrinsic geodesic equation of motion given by the following generalized metric-compatible connection form:

$$\Gamma^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma}(x,u) = \frac{1}{2}\, g^{\alpha\rho}\left(\frac{\partial g_{\rho\beta}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\rho} + \frac{\partial g_{\gamma\rho}}{\partial x^\beta}\right) + \\ + \frac{e}{2mc^2}\left(F_{\beta\gamma}u^\alpha - F^{\alpha\cdot}_{\cdot\beta}u_\gamma - F^{\alpha\cdot}_{\cdot\gamma}u_\beta\right) + S^{\alpha\cdot\cdot}_{\cdot\beta\gamma} - g^{\alpha\rho}\left(S_{\beta\rho\gamma} + S_{\gamma\rho\beta}\right). \quad (4.1)$$

The anti-symmetric electromagnetic field tensor $F_{\alpha\beta}$ is fully geometrized through the relation

$$F_{\alpha\beta} = 2\,\frac{mc^2}{e}\,\Gamma^\lambda_{[\alpha\beta]}\,u_\lambda\,, \quad (4.2)$$

whose interior structure is given by the geometrized Yang-Mills gauge field [5], here in terms of the *internal* material coordinates of $\mathrm{C}^3(t)$ as

$$F^i_{\alpha\beta} = -2\,h^i_\lambda\Gamma^\lambda_{[\alpha\beta]} = \frac{\partial A^i_\alpha}{\partial x^\beta} - \frac{\partial A^i_\beta}{\partial x^\alpha} + i\,\hat{g}\,\epsilon^{i\cdot\cdot}_{\cdot kl}A^k_\alpha A^l_\beta + 2Z^{i\cdot}_{\cdot k}A^k_{[\alpha}u_{\beta]}\,, \quad (4.3)$$

$$F_{\alpha\beta} = \frac{mc^2}{e}\,F^i_{\alpha\beta}\,u_i\,, \qquad \Omega^i_{[kl]} = \frac{1}{2}\,i\,\hat{g}\,\epsilon^{i\cdot\cdot}_{\cdot kl}\,, \quad (4.4)$$

where $A^i_\alpha = -h^i_\alpha$ is the gauge field strength (not to be confused with the angular momentum), $\hat{g}$ is a coupling constant, and $\epsilon^{i\cdot\cdot}_{\cdot kl}$ is the three-dimensional permutation tensor.

The material spin tensor $S^{\alpha\cdot\cdot}_{\cdot\beta\gamma}$ is readily identified here through the anti-symmetric part of the four-dimensional form of the extrinsic curvature $\varphi_{\alpha\beta}$ (i.e., the material vorticity $\omega_{\alpha\beta}$) of M^4:

$$S^{\alpha\cdot\cdot}_{\cdot\beta\gamma} = S^{\alpha\cdot}_{\cdot\beta}\, u_\gamma - S^{\alpha\cdot}_{\cdot\gamma}\, u_\beta \,, \tag{4.5}$$

$$S_{\alpha\beta} = \hat{s}\,\varphi_{[\alpha\beta]} = \hat{s}\,\omega_{\alpha\beta} = \frac{1}{2}\,\hat{s}\left(\nabla_\beta u_\alpha - \nabla_\alpha u_\beta\right), \tag{4.6}$$

where $\hat{s}$ is a constant spin coefficient, which can possibly be linked to the electric charge e, the mass m, and the speed of light in vacuum c, and hence to the Planck-Dirac constant $\hbar$ as well, such that we can express the connection form more compactly as

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\, g^{\alpha\rho}\left(\frac{\partial g_{\rho\beta}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\rho} + \frac{\partial g_{\gamma\rho}}{\partial x^\beta}\right) + \\ + \frac{e}{2mc^2}\left(F_{\beta\gamma}u^\alpha - F^{\alpha\cdot}_{\cdot\beta}\, u_\gamma - F^{\alpha\cdot}_{\cdot\gamma}\, u_\beta\right) - 2S^{\alpha\cdot}_{\cdot\beta}\, u_\gamma \,. \tag{4.7}$$

Therefore, owing to the fully intrinsic dynamics of the geometrized physical fields located in M^4, i.e.,

$$\frac{Du^\alpha}{ds} = u^\beta\,\nabla_\beta\, u^\alpha = 0\,, \tag{4.8}$$

we see that the following condition is naturally satisfied:

$$S_{\alpha\beta}\, u^\beta = 0 \tag{4.9}$$

in addition to the equation of motion

$$mc^2\left(\frac{du^\alpha}{ds} + \Delta^\alpha_{\beta\gamma}\, u^\beta u^\gamma\right) = eF^{\alpha\cdot}_{\cdot\beta}\, u^\beta, \tag{4.10}$$

where the usual connection coefficients are

$$\Delta^\alpha_{\beta\gamma} = \frac{1}{2}\, g^{\alpha\rho}\left(\frac{\partial g_{\rho\beta}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\rho} + \frac{\partial g_{\gamma\rho}}{\partial x^\beta}\right). \tag{4.11}$$

Meanwhile, from §2, we note that

$$\varphi_{\alpha\beta} = \nabla_\beta\, u_\alpha = h^i_\alpha\, h^k_\beta\, Z_{ik}\,, \tag{4.12}$$

$$Z_{ik} = -\,u_\alpha\,\nabla_k\, h^\alpha_i\,, \tag{4.13}$$

$$\varphi_{\alpha\beta}\, u^\alpha = 0\,, \qquad \varphi_{\alpha\beta}\, u^\beta = 0\,, \tag{4.14}$$

and so we (re-)obtain

$$\Omega^p_{ik} = h^p_\alpha \frac{\partial h^\alpha_i}{\partial Y^k} - h^p_\alpha \Gamma^\alpha_{\beta\gamma} h^\beta_i h^\gamma_k \,, \tag{4.15}$$

$$\Gamma^\alpha_{\beta\gamma} = h^\alpha_i \frac{\partial h^i_\beta}{\partial x^\gamma} - h^\alpha_p \Omega^p_{ik} h^i_\beta h^k_\gamma + \varphi_{\beta\gamma} u^\alpha + u^\alpha \frac{\partial u_\beta}{\partial x^\gamma} - \varphi^{\alpha\cdot}_{\cdot\gamma} u_\beta \,, \tag{4.16}$$

where the material connection of $\mathrm{C}^3(t)$ can be explicitly expressed as

$$\Omega^i_{kl} = \frac{1}{2}\, h^{ip} \left(\frac{\partial h_{pk}}{\partial Y^l} - \frac{\partial h_{kl}}{\partial Y^p} + \frac{\partial h_{lp}}{\partial Y^k} \right) + \frac{1}{2}\, i \hat{g}\, \epsilon^{i\cdot\cdot}_{\cdot kl} \,, \tag{4.17}$$

or, in other words,

$$\Omega^i_{kl} = \frac{1}{2}\, h^{ip} \left(\frac{\partial h_{pk}}{\partial x^l} - \frac{\partial h_{kl}}{\partial x^p} + \frac{\partial h_{lp}}{\partial x^k} \right) + \\ + \frac{1}{c^2} \left(D^i_k v_l - D_{kl} v^i + D^i_l v_k \right) + \frac{1}{2}\, i \hat{g}\, \epsilon^{i\cdot\cdot}_{\cdot kl} \,. \tag{4.18}$$

This way, we have also obtained the fundamental structural forms corresponding to the immediate structure of our geometric theory of chiral elasticity [6], which, to a certain extent, is capable of encompassing the elastodynamics of matter in our present theory, as represented by the material hypersurface $\mathrm{C}^3(t)$.

§5. A Machian monad model of the Universe. We shall now turn towards developing a particular pure monad model, i.e., one in which the Universe possesses absolute angular momentum such that matter arises entirely from the intrinsic inhomogeneity and anisotropy emerging from the non-orientability and discontinuity of the very geometry of the material hypersurface $\mathrm{C}^3(t)$ with respect to the background space-time M^4. This goes down to saying that the cosmos has neither "inside" nor "outside" as graphically outlined in [4], and that each point in space-time indeed possesses *intrinsic informational spin*, irrespective of whether or not its corresponding empirical constitution possesses extrinsic angular momentum.

Recall, from the previous section, that the anti-symmetric part of the material connection form is given by the complex expression

$$\Omega^i_{[kl]} = \frac{1}{2}\, i \hat{g}\, \epsilon^{i\cdot\cdot}_{\cdot kl} \,, \tag{5.1}$$

which displays the internal constitution of matter in terms of the gauge coupling constant $\hat{g}$. Now, the four-dimensional permutation tensor is readily given by

$$\epsilon_{ikl}\, u_\gamma = -\,\epsilon_{\alpha\beta\rho\gamma}\, h^\alpha_i\, h^\beta_k h^\rho_l\,, \tag{5.2}$$

i.e.,

$$\epsilon_{\alpha\beta\rho\gamma} = -\,\epsilon_{ikl}\, h^i_\alpha h^k_\beta h^l_\rho\, u_\gamma + a_{\alpha\beta\rho\gamma} + b_{\alpha\beta\rho\gamma} + c_{\alpha\beta\rho\gamma}\,, \tag{5.3}$$

$$\epsilon_{ikl} = -\,\epsilon_{\alpha\beta\rho\gamma}\, h^\alpha_i\, h^\beta_k\, h^\rho_l\, u^\gamma, \tag{5.4}$$

$$a_{\alpha\beta\rho\gamma} = \epsilon_{\mu\beta\rho\gamma}\, u^\mu u_\alpha\,, \tag{5.5}$$

$$b_{\alpha\beta\rho\gamma} = \epsilon_{\alpha\mu\rho\gamma}\, u^\mu u_\beta\,, \tag{5.6}$$

$$c_{\alpha\beta\rho\gamma} = \epsilon_{\alpha\beta\mu\gamma}\, u^\mu u_\rho\,. \tag{5.7}$$

We therefore see that

$$\Omega^i_{[kl]} = -\frac{1}{2}\; i\,\hat{g}\,\epsilon^{\alpha\cdot\cdot\cdot}_{\cdot\beta\rho\gamma}\, h^i_\alpha\, h^\beta_k\, h^\rho_l\, u^\gamma, \tag{5.8}$$

and, in particular, that

$$\nabla_m\, \Omega^i_{[kl]} = -\frac{1}{2}\; i\,\hat{g}\, h^i_\alpha\, h^\beta_k\, h^\rho_l\, h^\lambda_m\, \epsilon^{\alpha\cdot\cdot\cdot}_{\cdot\beta\rho\gamma}\, \varphi^{\gamma\cdot}_{\cdot\lambda}\,. \tag{5.9}$$

The spatial curvature giving rise to matter can now be written as

$$R^{i\cdot\cdot\cdot}_{\cdot jkl} = B^{i\cdot\cdot\cdot}_{\cdot jkl} + M^{i\cdot\cdot\cdot}_{\cdot jkl}\,, \tag{5.10}$$

$$B^{i\cdot\cdot\cdot}_{\cdot jkl} = \frac{\partial P^i_{jl}}{\partial Y^k} - \frac{\partial P^i_{jk}}{\partial Y^l} + P^m_{jl}\, P^i_{mk} - P^m_{jk}\, P^i_{ml}\,, \tag{5.11}$$

$$M^{i\cdot\cdot\cdot}_{\cdot jkl} = \hat{\nabla}_k\, C^i_{jl} - \hat{\nabla}_l\, C^i_{jk} + C^m_{jl}\, C^i_{mk} - C^m_{jk}\, C^i_{ml}\,, \tag{5.12}$$

$$P^i_{kl} = \frac{1}{2}\, h^{ip}\left(\frac{\partial h_{pk}}{\partial Y^l} - \frac{\partial h_{kl}}{\partial Y^p} + \frac{\partial h_{lp}}{\partial Y^k}\right) = \Delta^i_{kl}\,, \tag{5.13}$$

$$C^i_{kl} = \Omega^i_{[kl]} - h^{ip}\left(h_{km}\,\Omega^m_{[pl]} + h_{lm}\,\Omega^m_{[pk]}\right) = \Omega^i_{[kl]}\,, \tag{5.14}$$

where $\hat{\nabla}$ denotes covariant differentiation with respect to the symmetric connection form $P^i_{kl}(\Delta^i_{kl})$.

The special integrability conditions for our particular model of space-time geometry possessing absolute angular momentum will be given by

$$h^\beta_i\, \Delta^\alpha_{\beta\gamma} = 0\,, \tag{5.15}$$

$$h^\alpha_i\, h^\beta_j\, h^\rho_k\, h^\gamma_l \left(R_{\alpha\beta\rho\gamma} + \varphi_{\alpha\rho}\,\varphi_{\beta\gamma} - \varphi_{\alpha\gamma}\,\varphi_{\beta\rho}\right) = 0\,, \tag{5.16}$$

such that, explicitly,

$$\Omega^i_{kl} = h^i_\alpha\, \frac{\partial h^\alpha_k}{\partial Y^l} = h^i_\alpha\, \frac{{}^*\partial h^\alpha_k}{\partial x^l} = h^i_\alpha\left(\frac{\partial h^\alpha_k}{\partial x^l} + \frac{1}{c^2}\, v_l\, \frac{{}^*\partial h^\alpha_k}{\partial t}\right). \tag{5.17}$$

We therefore obtain

$$\Delta^{\alpha}_{\beta\gamma} = \frac{1}{2}\, u_{\beta}\, g^{\alpha\rho} \left(\frac{\partial u_{\rho}}{\partial x^{\gamma}} - \frac{\partial u_{\gamma}}{\partial x^{\rho}} + \frac{d g_{\rho\gamma}}{ds} \right) + \frac{1}{2}\, u_{\beta} \left(g_{\rho\gamma} \frac{\partial u^{\rho}}{\partial x_{\alpha}} - \frac{\partial u^{\alpha}}{\partial x^{\gamma}} \right), \quad (5.18)$$

i.e.,

$$\Delta^{\alpha}_{\beta\gamma} = \frac{1}{2}\, u_{\beta}\, g^{\alpha\rho} \frac{d g_{\rho\gamma}}{ds} + u_{\beta} \left(g^{\alpha\rho}\, \Gamma^{\sigma}_{[\rho\gamma]}\, u_{\sigma} + u^{\rho}\, \Gamma^{\alpha}_{[\rho\gamma]} \right) \quad (5.19)$$

such that the world-velocity u^{α} plays the role of a fundamental "metric vector".

This way, matter (material curvature), and hence inertia, arises purely from the segmental torsional (discontinuity) curvature as follows:

$$R^{i\cdots}_{\cdot jkl} = -h^{i}_{\alpha} \left[\frac{\partial}{\partial Y^{l}} \left(\frac{\partial h^{\alpha}_{j}}{\partial Y^{k}} \right) - \frac{\partial}{\partial Y^{k}} \left(\frac{\partial h^{\alpha}_{j}}{\partial Y^{l}} \right) \right], \quad (5.20)$$

i.e.,

$$R^{i\cdots}_{\cdot jkl} = -\frac{2}{c^{2}}\, h^{i}_{\alpha}\, A_{kl}\, \frac{{}^{*}\partial h^{\alpha}_{j}}{\partial t}\,, \quad (5.21)$$

where the angular momentum A_{ik} is given by

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_{k}}{\partial x^{i}} - \frac{\partial v_{i}}{\partial x^{k}} \right) + \frac{1}{2c^{2}} (F_{i}\, v_{k} - F_{k}\, v_{i}) - \\ - u_{\alpha} \left(c\, \Gamma^{\alpha}_{[ik]} + \frac{v_{i}}{\sqrt{g_{00}}}\, \Gamma^{\alpha}_{[0k]} + \frac{v_{k}}{\sqrt{g_{00}}}\, \Gamma^{\alpha}_{[i0]} \right), \quad (5.22)$$

$$F_{i} = \frac{1}{\sqrt{g_{00}}} \left(\frac{\partial \mathrm{w}}{\partial x^{i}} - \frac{\partial v_{i}}{\partial t} \right) + 2\, \frac{c^{2}}{\sqrt{g_{00}}}\, \Gamma^{\alpha}_{[0i]}\, u_{\alpha}\,, \quad (5.23)$$

$$v_{i} = -c\, \frac{g_{0i}}{\sqrt{g_{00}}} = -c\, u_{i}\,, \qquad u^{i} = 0\,, \quad (5.24)$$

$$u_{0} = \sqrt{g_{00}}\,, \qquad u^{0} = \frac{1}{\sqrt{g_{00}}}\,, \quad (5.25)$$

$$\mathrm{w} = c^{2} \left(1 - \sqrt{g_{00}} \right). \quad (5.26)$$

In this particular scheme, therefore, every constitutive object in the Universe spins in the topological sense of gaining informational spin from the very formation of matter itself. Inertia would then be a property of matter directly arising from this intrinsic mechanism of spin, which encompasses the geometric formation of all massive objects at any scale. This, in turn, subtly corresponds to the Machian conjecture of the inertia (mass) of an object being dependent on a distant, massive frame of reference (if not all other massive objects in the Universe). However, since our peculiar geometric mechanism here exists at every point of space-time, and in the topological background of things, the corresponding generation of inertia is simply more intrinsic than the initial Machian scheme. Accordingly, there is no need to invoke the existence of a distant galactic frame of reference, other than the general non-orientability and curvature-generating discreteness of the hypersurface representing matter.

§6. Conclusion. We have outlined a seminal sketch of a fully hydrodynamical geometric theory of space-time and fields, which might complement Zelmanov's chronometric formulation of the General Theory of Relativity. In our theory, chronometricity is particularly geometrized through the unique hydrodynamical nature of the asymmetric extrinsic curvature of the material hypersurface.

Following our previous works we have unified the gravitational and electromagnetic fields, with chromodynamics arising from the fully geometrized inner structure of the electromagnetic field, which is shown to be the Yang-Mills gauge field (appearing here in its generalized form). In the present work, it is interesting to note that the role of the non-Abelian gauge field (represented by its components, namely, A^i_α) is very naturally played by the projective chronometric structure (with components h^i_α), and so the inner constitution of elementary particles cannot be divorced from chronometricity at all.

In our approach to Mach's principle through a pure monad model possessing absolute angular momentum, the unique Kleinian topology of the Universe gives rise to inertia in terms of the non-orientable spin dynamics and discrete intrinsic geometry of the material hypersurface, rendering the respective generation of inertia both local and global (i.e., signifying, in a cosmological sense, scale-independence as well as intrinsic topological interdependence among "particulars" and "universals").

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